University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

GUANGZHAO ZHU

BSc (Central China Normal University, 2020)

"Algebraic Cycles on Products of Generically Smooth Quadrics"

Department of Mathematics and Statistics

Friday, December 9, 2022
10:00 A.M.
David Strong Building
Room C114

Supervisory Committee:
Dr. Stephen Scully, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Ryan Budney, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Zinovy Reichstein, Department of Mathematics, University of British Columbia

Chair of Oral Examination:
Dr. Simon Devereaux, Department of Mathematics and Statistics, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

In this thesis, we study rationality questions for algebraic cycles (modulo-2) on products of generically smooth smooth projective quadrics over fields of any characteristic. More specifically, let $X_1, \cdots, X_n$ be generically smooth projective quadrics over a field F with anisotropic totally singular part. We study the image $\overline{Ch}(X_1 \times \cdots \times X_n)$ of scalar-extension homomorphism $Ch(X_1 \times \cdots \times X_n) \to Ch((X_1 \times \cdots \times X_n)_{\bar{F}})$, where $\bar{F}$ denotes a fixed algebraic closure of F and Ch denotes the total Chow group modulo 2. This has been studied extensively in the case where $X_1, \cdots, X_n$ are smooth by A.Vishik, N.Karpenko, A.Merkurjev and others. Our goal is to extend the existing theory to include the case of singular but generically smooth quadrics in characteristic 2. Here we follow recent work of Karpenko, who has considered the special case where $X_1 = \cdots = X_n$.

First, we show that the image $\overline{Ch}(X_1 \times \cdots \times X_n)$ inherits a ring structure and an action of Steenrod operations from the mod-2 Chow ring of the smooth locus of $X_1 \times \cdots \times X_n$. Using the ring structure, we then introduce and study a composition of rational correspondences (modulo 2) for products of generically smooth projective quadrics, laying foundations for an investigation of non-totally singular quadratic forms in any characteristic by algebraic-geometric methods. In this direction, we introduce a new discrete invariant for such forms, which we call the *rational correspondence type*. This extends the *motivic decomposition type* previously defined by Vishik for non-degenerate forms. We extend several well-known results on Vishik's invariant to our more general setting. These include a number of restrictions imposed by splitting pattern invariants, as well as results that relate the rational correspondence types of different forms in situations where their associated quadrics can be related via suitable Chow correspondences. Using these results, we compute the rational correspondence type for certain families of forms, including generic forms of even-dimension, and so-called quasi-strongly excellent forms. In the final part of the thesis, we show that the deepest result of Vishik on motivic decomposition type, the so-called *excellent connections theorem*, remains valid for arbitrary non-totally singular forms of dimension at most 9. We also apply our methods to the study of a conjecture of Hoffmann and Laghribi on the classification of singular Pfister neighbours, and to the study of the isotropy behaviour of quadratic forms over function fields of quadrics.