On the ergodicity of a class of 1-dimensional probabilistic cellular automata with size-3 neighbourhoods

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Abstract

Let $\mathcal{S} = \{(p,q) \in [0,1]^2 : 0 < p + q \leqslant 1\}$. For any fixed $(p,q) \in \mathcal{S}$, we consider a 1-dimensional probabilistic cellular automaton (PCA) $F_{p,q}$, with alphabet $\mathcal{A} = \{0,1\}$, neighbourhood $\mathcal{N} = \{0,1,2\}$, and (stochastic) local update rules that are defined as follows. If $\eta = (\eta(n) : n \in \mathbb{Z})$ is a configuration in $\mathcal{A}^{\mathbb{Z}}$, then, given $\eta(n) = \eta(n+1) = \eta(n+2) = 0$, we have

$$F_{p,q}\eta(n) = \begin{cases} 0 & \text{with probability } p, \\ 1 & \text{with probability } 1-p, \end{cases}$$

and given $(\eta(n), \eta(n+1), \eta(n+2)) \in \mathcal{A}^3 \setminus \{(0,0,0)\}$, we have

$$F_{p,q}\eta(n) = \begin{cases} 0 & \text{with probability } 1-q, \\ 1 & \text{with probability } q. \end{cases}$$

We show that $F_{p,q}$ is ergodic if and only if the probability of draw in certain suitably defined percolation games on $\mathbb{Z}^2$ is 0. Next, we establish connections between these games and a different but related PCA, $\widehat{F}_{p,q}$, with alphabet $\hat{\mathcal{A}} = \{0, ?, 1\}$, that is usually referred to as the "envelope" to $F_{p,q}$. We employ the relatively unexplored technique of *weight functions* to show that the probability of appearance of the symbol ? is 0 under *every* stationary distribution for $\widehat{F}_{p,q}$, which in turn implies that the probability of draw is 0 in each of our games, for every $(p,q) \in \mathcal{S}$.