University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

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"Dynamical Classification of the Two-body and Hill's Lunar Problems with Quasi-homogeneous Potentials"

Department of Mathematics and Statistics

Thursday, June 30, 2022
3:00 P.M.
David Strong Building
Room C126

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Abstract

As seen in many examples, higher order correction added to the Newtonian potential often provides more realistic and accurate model in astrophysics. Such examples include the Schwarzschild potential ($U_{\text{Schwarzschild}}(r) = -\frac{A}{r} - \frac{B}{r^3}$, where r is the mutual distance and A, B are constants) and the Manev potential ($U_{\text{Manev}}(r) = -\frac{A}{r} - \frac{B}{r^2}$).

In the thesis, we study the two-body problem and Hill's lunar problem under quasi-homogeneous potentials.

The quasi-homogeneous two-body problem aims to study the interaction between two point particles under a prescribed potential in the form of $W(r) = U(r) + V(r) = -\frac{A}{r^a} - \frac{B}{r^b}$. It is well known that the number two serves as a threshold value for homogeneous $(U(r) \simeq -\frac{1}{r^a}$ N-body problems: one is able to observe significant difference regarding to the solution dynamics as the power of homogeneous potential exceeds two from below. This phenomenon remains observable for quasi-homogeneous potentials.

In the second chapter, we provide a complete characterization of the whole phase space of the quasi-homogeneous two-body problem in terms of global existence and singularity for all the possible$b > a > 0$. In particular, one is able to generalize the result of Manev two-body problem ($a = 1, b = 2$) to all the quasi-homogeneous potentials with$b = 2 > a$. Two techniques are presented: One is the variational method based on energy and the other is direct computation of collision time based on the integrability of two-body systems.

Hill's lunar equation Hill's lunar equation under the Newtonian or homogeneous potentials has been derived from the Hamiltonian of the three-body problem in a uniform rotating coordinate system with angular speed ω, by using symplectic scaling and heuristic arguments on various physical quantities.

Quasi-homogeneous Hill's lunar problem is the other focus of this thesis. In the third chapter, we first derive Hill's lunar equation for quasi-homogeneous potentials. We then fully characterize the phase space under some energy threshold using an indicator function. This energy threshold is characterized variationally. In particular, our results demonstrate the existence of "black hole effect" for $b > 2 > a$ and ω sufficiently large: invariant sets (in the

phase space) with non-zero Lebesgue measure that either contains global solutions and solutions with singularity are constructed, under and at some energy threshold. Next, we apply McGehee-type transform to study near-collision dynamics of the homogeneous Hill's lunar problem. We shall derive the asymptotic profile near collision for all the strong-force ($a > 2$) potentials.

Finally, in the last chapter, we conclude the advantages and disadvantages of the methods we have used through this thesis, as well as summarizing our future works.