



University
of Victoria

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

CONNOR WAGNER

BSc (University of Victoria, 2017)

“A New Survey on the Firefighter Problem”

Department of Mathematics and Statistics

Tuesday, August 24, 2021

8:30am (PDT)

Remote Defence

Supervisory Committee:

Dr. Gary MacGillivray, Department of Mathematics and Statistics,
University of Victoria (Co-Supervisor)

Dr. Kieka Mynhardt, Department of Mathematics and Statistics, UVic (Co-Supervisor)

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Abstract

Firefighter is a discrete-time dynamic process that models the spread of a virus or rumour through a network. The name "Firefighter" arises from the initial analogy being the spread of fire among the vertices of a graph. Given a graph G , the process begins at time $t = 0$ when one or more vertices of G spontaneously "catch fire". At each subsequent time step, a collection of $b \geq 1$ "firefighters" defend a set of vertices which are not burning, and then the fire spreads from each burning vertex to all of its undefended neighbours. There are many possible objectives one could have, for example minimizing the expected number of vertices burned when the fire breaks out at a random location or locations, finding the maximum number of vertices that can be saved from burning if the fire breaks out at known locations, minimizing the length of the process, or bounding the proportion of vertices that can be saved from burning. It is also possible to consider multiple objectives that may be in conflict. There are a great number of papers in the literature which address these, and other, issues in terms of computational complexity, algorithms, approximation, asymptotics, heuristics, and more.

The main purpose of this thesis is to survey developments on Firefighter and its variants which have appeared in the literature subsequent to a previous survey that appeared in 2009 [S. Finbow and G. MacGillivray. The firefighter problem: A survey of results, directions and questions. *Australas. J. Comb.*, 43, 2009]. It concludes with a list of open problems and future directions from the previous survey, annotated with references for papers that have made progress on those topics since then.