Notice of the Final Oral Examination for the Degree of Master of Science of

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BSc (Thompson River University, 2018)

“Switchable Homomorphisms of \((m, n)\)-Mixed Graphs”

Department of Mathematics and Statistics

Friday, August 20, 2021
2:00pm (PDT)
Remote Defence

Supervisory Committee:
Dr. Gary MacGillivray, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Richard Brewster, Department of Mathematics and Statistics, UVic (Co-Supervisor)

External Examiner:
Dr. Mark Siggers, Department of Mathematics, Kyungpook National University

Chair of Oral Examination:
Dr. David Blades, Department of Curriculum and Instruction, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies
Abstract

A \((m, n)\)-mixed graph is a mixed graph whose edge set is partitioned into \(m\) colours, and whose arc set is partitioned into \(n\) colours. Let \(G\) be a \((m, n)\)-mixed graph and \(\pi\) is a \((n + 2)\)-tuple of permutations from \(S_m \times S_n \times S'_2\). We define switching at a vertex \(v\) with respect to \(\pi\) as follows. Replace each edge \(vw\) of colour \(\phi\) by an edge \(vw\) of colour \(\alpha(\phi)\), and each arc \(vx\) of colour \(\phi\) by an arc \(\gamma(\phi)(vx)\) of colour \(B(\phi)\).

In this thesis, we study the complexity of the question: “Given a \((m, n)\)-mixed graph \(G\), is there a sequence of switches at vertices of \(G\) with respect to the fixed group \(\Gamma\) so that the resulting \((m, n)\)-mixed graph admits a homomorphism to a \((m, n)\)-mixed graph on 2 vertices.”

We show the following: (1) When restricted to \((m, 0)\)-mixed graphs \(H\) on at most 2 vertices, the \(\Gamma\)-switchable homomorphism decision problem is solvable in polynomial time; (2) for each \((0, n)\)-mixed graph \(H\), there is a \((2n, 0)\)-mixed graph such that the respective \(\Gamma\)-switchable homomorphism decision problems are polynomial equivalent; (3) For all \((m, n)\)-mixed graphs and groups, when \(H\) has at most 2 vertices, the decision problem is polynomial time solvable; (4) For a yes-instance, we can find in quadratic time a sequence of switches on \(G\) such that the resulting \((m, 0)\)-mixed graph admits a homomorphism to \(H\).

By proving (1)-(4), we complete a dichotomy theorem for the complexity of the \(\Gamma\)-switchable \(k\)-colouring problem and provide a step towards a dichotomy theorem for the complexity of \(\Gamma\)-switchable homomorphism decision problem.