University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BSc (Thompson River University, 2018)

"Switchable Homomorphisms of (*m, n*)-Mixed Graphs"

Department of Mathematics and Statistics

Friday, August 20, 2021
2:00pm (PDT)
Remote Defence

Supervisory Committee:
Dr. Gary MacGillivray, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Richard Brewster, Department of Mathematics and Statistics, UVic (Co-Supervisor)

External Examiner:
Dr. Mark Siggers, Department of Mathematics, Kyungpook National University

Chair of Oral Examination:
Dr. David Blades, Department of Curriculum and Instruction, UVic

Dr. Robin G. Hicks, Dean, Faculty of Graduate Studies

Abstract

A (m, n)-mixed graph is a mixed graph whose edge set is partitioned into m colours, and whose arc set is partitioned into n colours. Let G be a (m, n)-mixed graph and π is a (n + 2)-tuple of permutations from $S_m \times S_n \times S^n_2$. We define *switching at a vertex v with respect to π* as follows. Replace each edge vw of colour ϕ by an edge vw of colour $\alpha(\phi)$, and each arc vx of colour ϕ by an arc $\gamma_\phi(vx)$ of colour $B(\phi)$.

In this thesis, we study the complexity of the question: "Given a (m, n)-mixed graph G, is there a sequence of switches at vertices of G with respect to the fixed group Γ so that the resulting (m, n)-mixed graph admits a homomorphism to a (m, n)-mixedgraph on 2 vertices."

We show the following: (1) When restricted to (m, 0)-mixed graphs H on at most 2 vertices, the Γ -switchable homomorphism decision problem is solvable in polynomial time; (2) for each (0, n)-mixed graph H, there is a ($2n$, 0)-mixed graph such that the respective Γ -switchable homomorphism decision problems are polynomial equivalent; (3) For all (m, n)-mixed graphs and groups, when H has at most 2 vertices, the decision problem is polynomial time solvable; (4) For a yes-instance, we can find in quadratic time a sequence of switches on G such that the resulting (m, 0)-mixed graph admits a homomorphism to H.

By proving (1)-(4), we complete a dichotomy theorem for the complexity of the Γ -switchable k-colouring probem and provide a step towards a dichotomy theorem for the complexity of Γ -switchable homomorphism decision problem.