University of Victoria | Graduate Studies

Notice of the Final Oral Examination for the Degree of Doctor of Philosophy

of

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MSc (University of Victoria, 2013)
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"Sarvate-Beam Group Divisible Designs and Related Multigraph Decomposition Problems"

Department of Mathematics and Statistics

Friday, September 11, 2020
11:00 A.M.
Conducted Remotely

Supervisory Committee:
Dr. Peter Dukes, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic (Member)
Dr. Wendy Myrvold, Department of Computer Science, UVic (Outside Member)

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Dr. Ulrich Mueller, Department of Psychology, UVic

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Abstract

A design is a set of points *V* together with a set of subsets of *V* called *blocks*. A classic example of a design is a *balanced incomplete block design*, or BIBD. A BIBD is often called a (v, k, λ)-design when it has v points, blocks containing k points, and *index* λ. That is, each pair of points occurs together in a block exactly λ times. This 'balanced' condition can be replaced with other properties. An *adesign* is a design where we instead require every pair of points to occur a different number of times together in block. The number of times a specified pair of points occurs together is called the *pair frequency*.

We explore a special type of adesign called a Sarvate-Beam design, named after its founders D.G. Sarvate and W. Beam [22]. A *Sarvate-Beam design* is an adesign where the set of distinct pair frequencies cover an interval of consecutive nonnegative integers. Specifically we investigate the existence of Sarvate-Beam group divisible designs. A *group divisible design* or GDD, in the usual sense, is a set of points and blocks where the points are partitioned into subsets called *groups*. Any pair of points contained in a group have pair frequency zero and pairs of points from different groups have pair frequency one. When the groups are of equal size, the GDD is said to be *uniform*. A *Sarvate-Beam group divisible design*, or SBGDD, is a group divisible design where instead the set of pair frequencies of points from different groups are a set of distinct nonnegative consecutive integers.

The main result of this paper is to completely settle the existence question for uniform SBGDDs with blocks of size 3 where the smallest pair frequency, called the *starting frequency*, is zero. Higher starting frequencies, μ, are also considered and settled for all positive integers μ except in the case of 8 groups where a few unknown exceptions remain.

The relationship between the designs and graph decompositions is used and leads to some generalizations. Some ideas with matrices and linear programming are also explored and gives rise to related results.