University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

FLORA BOWDITCH

BSc (University of Victoria, 2017)

"Localized Structure in Graph Decompositions"

Department of Mathematics and Statistics

Friday, November 22, 2019
11:00 A.M.
David Strong Building
Room C130

Supervisory Committee:
Dr. Peter Dukes, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Esther Lamken, Mathematics and Statistics, Caltech

Chair of Oral Examination:
Dr. Daniela Constantinescu, Department of Mechanical Engineering, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

Let $v \in \mathbb{Z}^+$ and G be a simple graph. A *G-decomposition* of K_v is a collection $\mathcal{F} = \{F_1, F_2, \ldots, F_t\}$ of subgraphs of K_v such that every edge of K_v occurs in exactly one of the subgraphs and every graph $F_i \in \mathcal{F}$ is isomorphic to G. A G-decomposition of K_v is called *balanced* if each vertex of K_v occurs in the same number of copies of G. In 2011, Dukes and Malloch provided an existence theory for balanced G-decompositions of K_v. Shortly afterwards, Bonisoli, Bonvicini, and Rinaldi introduced *degree-* and *orbit-balanced* G-decompositions. Similar to balanced decompositions, these two types of G-decompositions impose a local structure on the vertices of K_v. In this thesis, we will present an existence theory for degree- and orbit-balanced G-decompositions of K_v. To do this, we will first develop a theory for decomposing K_v into copies of G when G contains coloured loops. This will be followed by a brief discussion about the applications of such decompositions. Finally, we will explore an extension of this problem where K_v is decomposed into a family of graphs. We will examine the complications that arise with families of graphs and provide results for a few special cases.