Notice of the Final Oral Examination
for the Degree of Master of Science
of
FLORA BOWDITCH
BSc (University of Victoria, 2017)

“Localized Structure in Graph Decompositions”

Department of Mathematics and Statistics

Friday, November 22, 2019
11:00 A.M.
David Strong Building
Room C130

Supervisory Committee:
Dr. Peter Dukes, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Esther Lamken, Mathematics and Statistics, Caltech

Chair of Oral Examination:
Dr. Daniela Constantinescu, Department of Mechanical Engineering, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

Let $v \in \mathbb{Z}^+$ and $G$ be a simple graph. A $G$-decomposition of $K_v$ is a collection $\mathcal{F} = \{F_1, F_2, \ldots, F_t\}$ of subgraphs of $K_v$ such that every edge of $K_v$ occurs in exactly one of the subgraphs and every graph $F_i \in \mathcal{F}$ is isomorphic to $G$. A $G$-decomposition of $K_v$ is called balanced if each vertex of $K_v$ occurs in the same number of copies of $G$. In 2011, Dukes and Malloch provided an existence theory for balanced $G$-decompositions of $K_v$. Shortly afterwards, Bonisoli, Bonvicini, and Rinaldi introduced degree- and orbit-balanced $G$-decompositions. Similar to balanced decompositions, these two types of $G$-decompositions impose a local structure on the vertices of $K_v$. In this thesis, we will present an existence theory for degree- and orbit-balanced $G$-decompositions of $K_v$. To do this, we will first develop a theory for decomposing $K_v$ into copies of $G$ when $G$ contains coloured loops. This will be followed by a brief discussion about the applications of such decompositions. Finally, we will explore an extension of this problem where $K_v$ is decomposed into a family of graphs. We will examine the complications that arise with families of graphs and provide results for a few special cases.