Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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MSc (Sichuan University, 2010)
BSc (Sichuan University, 2013)

"Central Configurations of the Curved *N*-Body Problem"

Department of Mathematics and Statistics

Tuesday, July 11, 201 7
1:30 P.M.
David Turpin Building
Room A144

Supervisory Committee:
Dr. Florin Diacu, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Slim Ibrahim, Department of Mathematics and Statistics, UVic (Member)
Dr. Alexandra Branzan Albu, Department of Electrical and Computer Engineering, UVic
(Outside Member)

External Examiner:
Dr. Zhifu Xie, Department of Mathematics and Statistics, University of Mississippi

Chair of Oral Examination:
Dr. Mohsen Akbari, Department of Mechanical Engineering, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

We extend the concept of central configurations to the *N*-body problem in spaces of nonzero constant curvature. Based on the work of Florin Diacu on relative equilibria of the curved N-body problem and the work of Smale on general relative equilibria, we find a natural way to define the concept of central configurations with the effective potentials. We characterize the ordinary central configurations as constrained critical points of the cotangent potential, which helps us to establish the existence of ordinary central configurations for any given masses. After these fundamental results, we study central configurations on $\mathbb{H}^2$, ordinary central configurations in S3, and special central configurations in $\mathbb{S}^3$ in three separate chapters. For central configurations on $\mathbb{H}^2$, we generalize the theorem of Moulton on geodesic central configurations, the theorem of Shub on the compactness of central configurations, the theorem of Conley on the index of geodesic central configurations, and the theorem of Palmore on the lower bound for the number of central configurations. We show that all three-body central configurations that form equilateral triangles must have three equal masses. For ordinary central configurations in $\mathbb{S}^3$, we construct a class of $\mathbb{S}^3$ ordinary central configurations. We study the geodesic central configurations of two and three bodies. Three-body non-geodesic ordinary central configurations that form equilateral triangles must have three equal masses. We also put into the evidence some other classes of central configurations. For special central configurations, we show that for any $N \geq 3$, there are masses that admit at least one special central configuration. We then consider the Dziobek special central configurations and obtain the central configuration equation in terms of mutual distances and volumes formed by the position vectors. We end the thesis with results concerning the stability of relative equilibria associated with 3-body special central configurations. We find that these relative equilibria are Lyapunov stable when confined to $\mathbb{S}^1$ and that they are linearly stable on $\mathbb{S}^2$ if and only if the angular momentum is bigger than a certain value determined by the configuration.