From Probability to the Gambler's Fallacy

Introduction to the theme

> *"It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge … The most important questions of life are, for the most part, really only problems of probability."*
> ~ Pierre Simon, Marquis de LaPlace, *Theorie Analytique des Probabilites*, 1812.

Gambling is a human activity that can involve probability calculations. Knowing how to calculate the probability of an event may help the gambler make informed decisions.

However, the mathematics of gambling can be compromised in at least three different ways. First, as in the real world, many times the conditions for accurately calculating the probability will not be available – outcomes may not be completely random and we may not know all the factors that influence the outcome (e.g., when betting on the outcome of a sporting event). Second, gambling can be exciting and stimulating because it involves taking a chance. We may get a rush out of that and choose to take risks even when we know the odds are stacked against us. Third, we may be influenced by a fallacy or misconception when calculating or estimating probability.

This lesson cannot make someone a more successful gambler. But it can help students develop an understanding of probability and its usefulness and limitations relative to gambling.

Instructional strategies

1. Have students, individually, in groups or as a class, test their probability ability at Game Sense. Or you can use the similar paper-based exercise included with this lesson. The answer key to the latter is: 1-b, 2-e, 3-f, 4-a, 5-c, 6-i, 7-h, 8-d, 9-g.
2. Go over the basic concepts of probability with the students and then have them work in small groups to solve the probability problems. Discuss the problems and their solutions as a class (see student handout).

   An **elementary event** is a single event, like rolling a 3 on a dice – in mathematical terms this event is expressed by the set {3}.

   A **compound event** is made up of multiple elementary events, like rolling a 3 and a 5 when rolling two dice – in this case the set would be {3, 5}.

   Probability is the likelihood that a given event will occur expressed as the ratio of the number of favourable outcomes divided by the number of possible outcomes for a given process. Hence, the probability of rolling a 4 on any roll of a single dice is 1/6 or about 0.17 or 17%.

   Randomness refers to a condition in which each outcome of a process has the same probability of occurring. Randomness does not mean that each outcome will occur the same number of times, only that each outcome has the same chance of occurring at any particular time.

   Combined probability applies to situations where multiple processes are involved, like rolling two dice (one red, one green). The probability of rolling a 3 on the red dice and a 5 on the green dice is a combination of the probability of each of the two events. This is expressed mathematically as 1/6 x 1/6 = 1/36.

Independent events do not in any way impact each other. In the above example, the outcome of rolling the red dice does not impact the outcome of rolling the green dice. The probability is thus calculated as a simple multiplication as above.

Dependent events influence each other's outcomes in some way. For example, imagine cards being dealt face up to two players from a shuffled deck (random order). The first player has a 4/52 (0.077) chance of getting an ace as the first card. If the first player got an ace, the second player has a 3/51 (0.059) chance of getting an ace. If the first player did not get an ace, the second player has a 4/51 (0.078) chance of getting an ace. The probability of the second event depends on the outcome of the first event.

3. Problems

a. If you toss a coin nine times and it comes up heads seven times and tails twice, what is the probability that it will be tails on the tenth throw?

b. If you toss three coins, what is the probability that only one will come up heads? Explain you answer.

c. If you toss two coins, what is the probability that they will both be heads? What is the probability that they will both be tails? What is the probability that one will be heads and the other tails? Explain.

After discussing this problem you could display the Tossing Coins Experiment and set the conditions to 2 coins and 40 throws. Hit the "toss coins" button several times and compare the result to what would be expected. You can change the number of throws and again hit the "toss coins" button several times. Ask students for their observations on what is happening. Note that probability does not predict what will happen only what is likely to happen.

d. If you throw two dice, what is the probability that:

i. Either dice is 3
ii. Neither dice is 3
iii. Both dice are 3
iv. At least one dice isn't 3
v. The highest dice is 3
vi. Dice are 3 and 5
vii. At least one dice is either 3 or 5

1-1	1-2	1-3	1-4	1-5	1-6
2-1	2-2	2-3	2-4	2-5	2-6
3-1	3-2	3-3	3-4	3-5	3-6
4-1	4-2	4-3	4-4	4-5	4-6
5-1	5-2	5-3	5-4	5-5	5-6
6-1	6-2	6-3	6-4	6-5	6-6

Hint: When solving 2-dice problems, using a 6x6 grid is often helpful in identifying the number of favourable outcomes.

e. Complete the following table with the probability that the sum of two dice will be:

2	3	4	5	6	7	8	9	10	11	12

After discussing this problem, you could display the Throwing Dice Experiment and set the conditions to 2 dice and 50 throws. Hit the "throw dice" button and compare the results to the actual probabilities. Do this several times and then change the number of throws and repeat. Ask students for their observations on what is happening. Why are the actual results sometimes quite different than the probability? Try to have students use the concepts introduced earlier to explain this difference.

f. If there are 6 blue balls and 4 green balls in a tumbler, what is the probability that the first two balls to drop will be blue? That they would both be green?

g. If batter A has faced pitcher B 15 times before and has 3 hits against B and comes to the plate twice while B is on the mound in the current game, what is the probability that A will get a hit off of B in this game? How confident would you be about predicting the outcome? Explain.

4. Based on the concepts of probability studied in this lesson, facilitate a discussion of the following common misconceptions.

   a. Things will always even out.
   Probability does not predict what will happen only what is likely to happen.

   b. If a number hasn't come up, it's due. If heads has occurred too often, tails is due.
   This is the classic gambler's fallacy and assumes the events are dependent when they are, in fact, independent.

   c. If a number comes up too often, there must be a bias.
   Biases do sometimes occur (e.g., faulty equipment, loaded dice), but more often an apparent bias is just a random fluke that does not provide any predictive power for what will happen next.

Gambling literacy

Big ideas

- Gambling can be a fun recreational activity but can also lead to significant harm
- We can learn how to control gambling by examining the different ways people have thought about it, engaging in critical self-reflection and listening to each other

Competencies

- Consider dominant social discourses and assess their impact on the distribution of risk and benefit associated with gambling
- Explore and appreciate the diverse cognitive, social, emotional and physical factors that impact gambling behaviour
- Develop personal and social skills to reflect on and manage personal behaviour and choices related to gambling

For a complete look at the gambling literacy competencies, as defined by the Centre for Addictions Research of BC, see: www.uvic.ca/research/centres/carbc/assets/docs/iminds/hs-gambling-curriculum.pdf)

Links to Curriculum

First Peoples' principles of learning

- Learning is holistic, reflexive, reflective, experiential, and relational (focused on connectedness, on reciprocal relationships, and a sense of place)
- Learning involves recognizing the consequences of one's actions
- Learning is embedded in memory, history, and story

Mathematics 9

Big ideas

- Analyzing the validity, reliability, and representation of data enables us to compare and interpret

Competencies

- Inductively and deductively reason and use logic to explore, make connections, predict, analyze, generalize, and make conclusions
- Develop and apply mental math strategies and estimate amounts and outcomes
- Develop, construct, and apply mathematical understanding through play, inquiry, and problem solving
- Engage in problem-solving experiences that are connected to place, story, and cultural practices relevant to the local community
- Visualize and describe the mathematical concepts
- Explore, apply, and connect concepts to each other, to other disciplines, and to the real world
- Use mathematical arguments to support personal choices and anticipate consequences

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