From Probability to the Gambler's Fallacy

"It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge … The most important questions of life are, for the most part, really only problems of probability."

~ Pierre Simon, Marquis de LaPlace, *Theorie Analytique des Probabilites*, 1812.

Gambling is a human activity that can involve probability calculations. Knowing how to calculate the probability of an event may help the gambler make informed decisions.

However, the mathematics of gambling can be compromised in several ways. First, as in the real world, many times the conditions for accurately calculating the probability will not be available – outcomes may not be completely random and we may not know all the factors that influence the outcome (e.g., when betting on the outcome of a sporting event). Second, gambling can be exciting and stimulating because it involves taking a chance. We may get a rush out of that and choose to take risks even when we know the odds are stacked against us. Third, we may be influenced by a fallacy or misconception when calculating or estimating probability.

This lesson cannot make you a more successful gambler, but it can help you develop an understanding of probability and its usefulness and limitations relative to gambling.

Basic concepts of probability

An **elementary event** is a single event, like rolling a 3 on a dice – in mathematical terms this event is expressed by the set {3}.

A **compound event** is made up of multiple elementary events, like rolling a 3 and a 5 when rolling two dice – in this case the set would be {3, 5}.

Probability is the likelihood that a given event will occur expressed as the ratio of the number of favourable outcomes divided by the number of possible outcomes for a given process. Hence, the probability of rolling a 4 on any roll of a single dice is 1/6 or about 0.17 or 17%.

Randomness refers to a condition in which each outcome of a process has the same probability of occurring. Randomness does not mean that each outcome will occur the same number of times, only that each outcome has the same chance of occurring at any particular time.

Combined probability applies to situations where multiple processes are involved, like rolling two dice (one red, one green). The probability of rolling a 3 on the red dice and a 5 on the green dice is a combination of the probability of each of the two events. This is expressed mathematically as
$1/6 \times 1/6 = 1/36$.

Independent events do not in any way impact each other. In the above example, the outcome of rolling the red dice does not impact the outcome of rolling the green dice. The probability is thus calculated as a simple multiplication as above.

Dependent events influence each other's outcomes in some way. For example, imagine cards being dealt face up to two players from a shuffled deck (random order). The first player has a 4/52 (0.077) chance of getting an ace as the first card. If the first player got an ace, the second player has a 3/51 (0.059) chance of getting an ace. If the first player did not get an ace, the second player has a 4/51 (0.078) chance of getting an ace. The probability of the second event depends on the outcome of the first event.

Problems

1. If you toss a coin nine times and it comes up heads seven times and tails twice, what is the probability that it will be tails on the tenth throw?
2. If you toss three coins, what is the probability that only one will come up heads? Explain you answer.
3. If you toss two coins, what is the probability that they will both be heads? What is the probability that they will both be tails? What is the probability that one will be heads and the other tails? Explain.
4. If you throw two dice, what is the probability that:
    a. Either dice is 3
    b. Neither dice is 3
    c. Both dice are 3
    d. At least one dice isn't 3
    e. The highest dice is 3
    f. Dice are 3 and 5
    g. At least one dice is either 3 or 5

1-1	1-2	1-3	1-4	1-5	1-6
2-1	2-2	2-3	2-4	2-5	2-6
3-1	3-2	3-3	3-4	3-5	3-6
4-1	4-2	4-3	4-4	4-5	4-6
5-1	5-2	5-3	5-4	5-5	5-6
6-1	6-2	6-3	6-4	6-5	6-6

Hint: When solving 2-dice problems, using a 6x6 grid is often helpful in identifying the number of favourable outcomes.

5. Complete the following table with the probability that the sum of two dice will be:

2	3	4	5	6	7	8	9	10	11	12

6. If there are 6 blue balls and 4 green balls in a tumbler, what is the probability that the first two balls to drop will be blue? That they would both be green?
7. If batter A has faced pitcher B 15 times before and has 3 hits against B and comes to the plate twice while B is on the mound in the current game, what is the probability that A will get a hit off of B in this game? How confident would you be about predicting the outcome? Explain.

Discussion

Based on the concepts of probability studied in this lesson, discuss the following.

a. Things will always even out.

b. If a number hasn't come up, it's due. If heads has occurred too often, tails is due.

c. If a number comes up too often, there must be a bias.