Coordinate transformation **Advanced Microscopy Facility**

This document describes the calculations necessary to transform a point of interest P between coordinate systems, when moving specimens between instruments. The described method is a Helmert Transformation, in two dimensions. Depending on the situation, this transformation could be used between the FB-2100 and S-4800 or vice versa, or even between different setups on the same machine, at different times.

This method requires two reference points to be created on the specimen, A and B. These could be Sharpie dots, FIB fabricated marks, or the like.

The typical use of the transformation would be:

1. Create two reference marks A and B on the specimen.
2. Use the FIB stage to move the specimen and determine the [x y] coordinates of A and B.
3. Move to your specified location(s) and fabricate the structures, recording the [x y] coordinates of each fabrication point P.
4. Move the specimen to the SEM (or to the FIB, at a later date).
5. Find the reference marks A and B, and record their [x y] coordinates.
6. Use this information along with the function described below to determine the [x y] coordinates of P, in the new system.
7. Translate the stage to the new position, and the fabrication point will be close by.

Initial coordinate system:

[A_x A_y], [B_x B_y], [P_x P_y]

Coordinate system to be transformed into:

[a_x a_y], [b_x b_y], [p_x p_y]

where [p_x p_y] are the computed coordinates of P in the new system.

The steps in the calculation are as follows:

1. Move P such that A is at the origin:

$$p_x = P_x - A_x$$
$$p_y = P_y - A_y$$

2. Use points A and B in the two systems to determine the rotation of the systems. $\emptyset$ is the angle between A and B in the initial system and θ is the angle between A and B in the new system.

$$\emptyset = \tan^{-1}\frac{B_y - A_y}{B_x - A_x}$$

$$\theta = \tan^{-1}\frac{b_y - a_y}{b_x - a_x}$$

In this construction, point A is assumed to be the origin. The arctangent function returns an angle between -90° and 90°, so it is essential to determine which quadrant point B is in, relative to point A. Based on the correct quadrant (determined by looking at the coordinates of B relative to A), it may be necessary to add 180° to the computed angle(s) before moving to the next step.

3. Rotate P about A (which is at the origin), using a rotation matrix, where $\alpha = \theta - \emptyset$:

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

Which expands to:

$$p_x = P_x \cos(\alpha) - P_y \sin(\alpha)$$
$$p_y = P_x \sin(\alpha) + P_y \cos(\alpha)$$

4. Determine scale factor F between coordinate systems. This takes into account different measurement units between systems:

$$F = \frac{\sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}}{\sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}}$$

This scale factor is applied to [p_x p_y] after step (3).

5. The next step is to apply a translation, to move P such that the origin is moved to [ax ay]:

$$p_x = P_x + a_x$$
$$p_y = P_y + a_y$$

6. When you combine all these steps into a single function, you get the following:

$$p_x = F\left[(P_x - A_x)\cos(\theta - \emptyset) - (P_y - A_y)\sin(\theta - \emptyset)\right] + a_x$$

$$p_y = F\left[(P_x - A_x)\sin(\theta - \emptyset) + (P_y - A_y)\cos(\theta - \emptyset)\right] + a_y$$