Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BSc (University of Victoria, 2017)

“Chromatic Polynomials of Mixed Graphs”

Department of Mathematics and Statistics

Wednesday, August 21, 2019
11:00 A.M.
Clearihue Building
Room B007

Supervisory Committee:
Dr. Gary MacGillivray, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Peter Dukes, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Danielle Cox, Department of Mathematics, Mount Saint Vincent University

Chair of Oral Examination:
Dr. Michel Lefebvre, Department of Physics and Astronomy, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

Let $G = (V, A, E)$ be a mixed graph and $c_o : V \to \{1, 2, \ldots, \lambda\}$ a function such that $c_o$ is a proper colouring of the underlying graph, $Und(G)$, and $c_o(u) \neq c_o(y)$ when $c_o(v) = c_o(x)$, for every pair of arcs $(u, v)$ and $(x, y)$. Such a function is called a proper oriented colouring of $G$. The number of proper oriented $\lambda$-colourings of $G$, denoted $f_o(G, \lambda)$, is a polynomial in $\lambda$. We call $f_o(G, \lambda)$ the mixed-chromatic polynomial of $G$.

In this thesis we will first present the basic theory of the mixed-chromatic polynomial. This theory will include computational tools and results concerning the coefficients of $f_o(G, \lambda)$. Next, we will consider the question of chromatic uniqueness and invariance of mixed graphs. Lastly, we reformulate a contract-delete recurrence for chromatic polynomials in order to enumerate various colourings, such as $k$–frugal $\lambda$–colourings.