University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

GRAHAM QUEE

BSc (University of Victoria, 2017)

"Ramp Approximations of Finitely Steep Sigmoid Control Functions in Soft-Switching ODE Networks"

Department of Mathematics and Statistics

Thursday, April 11, 2019
9:30 A.M.
Clearihue Building
Room B019

<u>Supervisory Committee:</u>
Dr. Roderick Edwards, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Junling Ma, Department of Mathematics and Statistics, UVic (Member)

<u>External Examiner:</u>
Dr. Tomas Gedeon, Mathematical Sciences, Montana State University

<u>Chair of Oral Examination:</u>
Dr. Michael McGuire, Department of Electrical and Computer Engineering, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

In models for networks of regulatory interactions in biological molecules, the sigmoid relationship between concentration of regulating bodies and the production rates they control has lead to the use of continuous time ‘switching’ ordinary differential equations (ODEs), sometimes referred to as Glass networks. These Glass networks are the result of a simplifying assumption that the switching behaviour occurs instantaneously at particular threshold values. Though this assumption produces highly tractable models, it also causes analytic difficulties in certain cases due to the discontinuities of the system, such as non-uniqueness. In this thesis we explore the use of ‘ramp’ functions as an alternative approximation to the sigmoid, which restores continuity to the ODE and removes the assumption of infinitely fast switching by linearly interpolating the focal point values used in a corresponding Glass network. A general framework for producing a ramp system from a certain Glass network is given. Solutions are explored in two dimensions, and then in higher dimensions under two different restrictions. Periodic behaviour is explored in both cases using mappings between threshold boundaries. Limitations in these method are explored, and a general proof of the existence of periodic solutions in negative feedback loops is given.