Notice of the Final Oral Examination for the Degree of Doctor of Philosophy

of

CHIRS BRUCE

BSc (University of Victoria, 2014)

“C*-algebras from actions of congruence monoids”

Department of Mathematics and Statistics

Thursday, April 16, 2020
10:00 A.M.
Remote Defence

Supervisory Committee:
Dr. Marcelo Laca, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Ian Putnam, Department of Mathematics and Statistics, UVic (Member)
Dr. Michel Lefebvre, Department of Physics and Astronomy, UVic (Outside Member)

External Examiner:
Dr. Judith A. Packer, Department of Mathematics, University of Colorado at Boulder

Chair of Oral Examination:
Dr. David Berg, Department of Chemistry, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

We initiate the study of a new class of semigroup C*-algebras arising from number theoretic considerations; namely, we generalize the construction of Cuntz, Deninger, and Laca by considering the left regular C*-algebras of $ax + b$-semigroups from actions of congruence monoids on rings of algebraic integers in number fields. Our motivation for considering actions of congruence monoids comes from class field theory and work on Bost–Connes type systems. We give two presentations and a groupoid model for these algebras, and establish a faithfulness criterion for their representations. We then explicitly compute the primitive ideal space, give a semigroup crossed product description of the boundary quotient, and prove that the construction is functorial in the appropriate sense. These C*-algebras carry canonical time evolutions, so that our construction also produces a new class of C*-dynamical systems. We classify the KMS (equilibrium) states for this canonical time evolution, and show that there are several phase transitions whose complexity depends on properties of a generalized ideal class group. We compute the type of all high temperature KMS states, and consider several related C*-dynamical systems.