Notice of the Final Oral Examination
for the Degree of Master of Science

of

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BSc (University of Victoria, 2017)

“Localized Structure in Graph Decompositions”

Department of Mathematics and Statistics

Friday, November 22, 2019
11:00 A.M.
David Strong Building
Room C130

Supervisory Committee:
Dr. Peter Dukes, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Esther Lamken, Mathematics and Statistics, Caltech

Chair of Oral Examination:
Dr. Daniela Constantinescu, Department of Mathematics and Statistics, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

Let \( v \in \mathbb{Z}^+ \) and \( G \) be a simple graph. A \( G \)-decomposition of \( K_v \) is a collection \( \mathcal{F} = \{F_1, F_2, ..., F_t\} \) of subgraphs of \( K_v \) such that every edge of \( K_v \) occurs in exactly one of the subgraphs and every graph \( F_i \in \mathcal{F} \) is isomorphic to \( G \). A \( G \)-decomposition of \( K_v \) is called balanced if each vertex of \( K_v \) occurs in the same number of copies of \( G \). In 2011, Dukes and Malloch provided an existence theory for balanced \( G \)-decompositions of \( K_v \). Shortly afterwards, Bonisoli, Bonvicini, and Rinaldi introduced degree- and orbit-balanced \( G \)-decompositions. Similar to balanced decompositions, these two types of \( G \)-decompositions impose a local structure on the vertices of \( K_v \). In this thesis, we will present an existence theory for degree- and orbit-balanced \( G \)-decompositions of \( K_v \). To do this, we will first develop a theory for decomposing \( K_v \) into copies of \( G \) when \( G \) contains coloured loops. This will be followed by a brief discussion about the applications of such decompositions. Finally, we will explore an extension of this problem where \( K_v \) is decomposed into a family of graphs. We will examine the complications that arise with families of graphs and provide results for a few special cases.