University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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"Relative Equilibria in The Curved *N*-Body Problem"

Department of Mathematics and Statistics

Thursday, August 30, 2018
12:00 P.M.
Clearihue Building
Room B017

Supervisory Committee:
Dr. Slim Ibrahim, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Ernesto Perez Chavela, Department of Mathematics and Statistics, UVic (Co-Supervisor)
Dr. Mihai Sima, Department of Electrical and Computer Engineering, UVic (Outside Member)

External Examiner:
Dr. Jamie Burgos-Garcia, Physics and Mathematics, Universidad Autónoma Metropolitana

Chair of Oral Examination:
Dr. Henning Struchtrup, Department of Mechanical Engineering, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

We consider the curved N-body problem, $N > 2$, in spaces of constant Gaussian curvature $\kappa \neq 0$; i.e., on spheres $\mathbb{S}^2_\kappa$, for $\kappa > 0$, and on hyperbolic manifolds $\mathbb{H}^2_\kappa$, for $\kappa < 0$. Our goal is to define and study relative equilibria, which are orbits whose mutual distances remain constant during the motion. We find new relative equilibria in the curved N-body problem for $N = 4$, and see whether bifurcations occur when passing through $\kappa = 0$. After obtaining a criterion for the existence of quadrilateral configurations on the equator of the sphere, we study two restricted 4-body problems: One in which two masses are negligible, and the second in which only one mass is negligible. In the former we prove the evidence for square-like relative equilibria, whereas in the latter we discuss the existence of kite-shaped relative equilibria.

We will further study the 5-body problem on 2-dimensional surfaces of constant curvature, with four of the masses arranged at the vertices of a square, and the fifth mass at the north pole of $\mathbb{S}^2_\kappa$, when the curvature is positive, it is shown that relative equilibria exists when the four masses at the vertices of the square are either equal or two of them are infinitesimal, such that they do not affect the motion of the remaining three masses. In the hyperbolic case, $\kappa < 0$, there exist two values for the angular velocity which produce negative elliptic relative equilibria when the masses at the vertices of the square are equal. We also show that the solutions with non-equal masses do not exist in $\mathbb{H}^2_\kappa$. Based on the work of Florin Diacu on the existence of relative equilibria for 3-body problem on the equator of $\mathbb{S}^2_\kappa$, we investigate the motion of more than three bodies. Furthermore, we study the motion of the negative curved 2- and 3-centre problems on the Poincaré upper semi-plane model. Using this model, we prove that the 2-centre problem is integrable, and we study the dynamics around the equilibrium point. Further, we analyze the singularities of the 3-centre problem due to the collision; i.e., the configurations for which at least two bodies have identical coordinates.