Notice of the Final Oral Examination
for the Degree of Master of Science
of

CLINT SEINEN

BASc (University of British Columbia, 2013)

“A Fast and Efficient Solver for Viscous-Plastic Sea Ice Dynamics”

Department of Mathematics and Statistics

Friday, September 15, 2017
9:00 A.M.
David Strong Building
Room C116

Supervisory Committee:
Dr. Boualem Khouider, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Junling Ma, Department of Mathematics and Statistics, UVic (Member)

ExternalExaminer:
Dr. Ben Nadler, Department of Engineering, UVic

Chair of Oral Examination:
Dr. Verena Tunnicliffe, Department of Biology, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

Sea ice plays a key role in the global climate system. Indeed, through the albedo effect it reflects significant solar radiation away from the oceans, while it also plays a key role in the momentum and heat transfer between the atmosphere and ocean by acting as an insulating layer between the two. Furthermore, as more sea ice melts due to climate change, additional fresh water is released into the upper oceans, affecting the global circulation of the ocean as a whole. While there has been significant effort in recent decades, the ability to simulate sea ice has lagged behind other components of the climate system and most Earth System Models fail to capture the observed losses of Arctic sea ice, which is largely attributed to our inability to resolve sea ice dynamics. The most widely accepted model for sea ice dynamics is the Viscous-Plastic (VP) rheology, which leads to a very non-linear set of partial differential equations that are known to be intrinsically difficult to solve numerically. This work builds on recent advances in solving these equations with a Jacobian-Free Newton-Krylov (JFNK) solver. We present an improved JFNK solver, where a fully second order discretization is achieved via the Crank Nicolson scheme and consistency is improved via a novel approach to the rheology term. More importantly, we present a significant improvement to the Jacobian approximation used in the Newton iterations, and partially form the action of the matrix by expressing the linear and nearly linear terms in closed form and approximating the remaining highly non-linear term with a second order approximation of its Gateaux derivative. This is in contrast with the previous approach which used a first order approximation for the Gateaux derivative of the whole functional. Numerical tests on synthetic equations confirm the theoretical convergence rate and demonstrate the drastic improvements seen by using a second order approximation in the Gateaux derivative. To produce a fast and efficient solver for VP sea ice dynamics, the improved JFNK solver is then coupled with a nonoscillatory, central differencing scheme for transporting sea ice as well as a novel method for tracking the ice domain using a level set method. Two idealized test cases are then presented and simulation results discussed, demonstrating the solver’s ability to efficiently produce Viscous-Plastic, physically motivated solutions.