

Elastica as a dynamical system

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Applied Mathematics Seminar
University of Victoria
September 22, 2015

Joint work with
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Abstract

Elastica is a curve that minimizes the integral over its length of the square of the curvature,

$$\int \kappa^2 ds.$$

Planar elasticae were considered at least as early as 1694 by James Bernoulli. However, it was not until about 1742 that Daniel Bernoulli convinced Euler to solve the problem by using the isoperimetric method. Exhaustive results were then published by Euler in 1744.

Euler's results were so comprehensive that the study of elastica remained somewhat dormant until taken up again by Max Born in his 1906 PhD thesis. Since then, many reseseachers explored interesting problems posed by elastica; see the technical report “*The elastica, a mathematical history*” by R. Levien, University of California, Berkeley, Aug 2008.

A significant motivation for this paper is to see what results could be obtained by a more pedestrian use of the techniques developed for mechanics like conservation laws, reduction of symmetry, solving the reduced equations and reconstruction. In order to be able to treat elastica as a mechanical system, we interpret the independent variable as “time”. In other words, we solve equations of motion of a particle evolving along an elastica.