University of Victoria | Graduate Studies

Notice of the Final Oral Examination for the Degree of Doctor of Philosophy

of

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"Stochastic Growth Models"

Department of Mathematics and Statistics

Monday May 11, 2015
2:00 P.M.
David Turpin Building
Room A144

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Abstract

This thesis is concerned with certain properties of stochastic growth models. A stochastic growth model is a model of infection spread, through a population of individuals, that incorporates an element of randomness. The models we consider are variations on the contact process, the simplest stochastic growth model with a recurrent infection.

Three main examples are considered. The first example is a version of the contact process on the complete graph that incorporates dynamic monogamous partnerships. To our knowledge, this is the first rigorous study of a stochastic spatial model of infection spread that incorporates some form of social dynamics. The second example is a non-monotonic variation on the contact process, taking place on the one-dimensional lattice, in which there is a random incubation time for the infection. Some techniques exist for studying non-monotonic particle systems, specifically models of competing populations [38] [12]. However, ours is the first rigorous study of a non-monotonic stochastic spatial model of infection spread. The third example is an additive two stage contact process, together with a general duality theory for multi-type additive growth models. The two-stage contact process is first introduced in [30], and several open questions are posed, most of which we have answered. There are many examples of additive growth models in the literature [27] [16] [30] [49], and most include a proof of existence of a dual process, although up to this point no general duality theory existed.

In each case there are three main goals. The first is to identify a phase transition with a sharp threshold or “critical value" of the transmission rate, or a critical surface if there are multiple parameters. The second is to characterize either the invariant measures if the population is infinite, or to characterize the metastable behaviour and the time to extinction of the disease, if the population is finite. The final goal is to determine the asymptotic behaviour of the model, in terms of the invariant measures or the metastable states.

In every model considered, we identify the phase transition. In the first and third examples we show the threshold is sharp, and in the first example we calculate the critical value as a rational function of the parameters. In the second example we cannot establish sharpness due to the lack of monotonicity. However, we show there is a phase transition within a range of transmission rates that is uniformly bounded away from zero and infinity, with respect to the incubation time.

For the partnership model, we show that below the critical value, the disease dies out within $C \log N$ time for some $C > 0$, where N is the population size. Moreover we show that above the critical value, there is a unique metastable proportion of infectious individuals that persists for at least $e^{\gamma N}$ time for some $\gamma > 0$. At the critical value, with

quite a bit of additional analysis, we show the extinction time for the infection is of order $\sqrt{N/\log\log N}$.

For the incubation time model, we use a block construction, with a carefully chosen good event to circumvent the lack of monotonicity, in order to show the existence of a phase transition. This technique also guarantees the existence of a non-trivial invariant measure. Due to the lack of additivity, the identification of all the invariant measures is not feasible. However, we are able to show the following is true. By rescaling time so that the average incubation period is constant, we obtain a limiting process as the incubation time tends to infinity, with a sharp phase transition and a well-defined critical value. We can then show that as the incubation time approaches infinity (or zero), the location of the phase transition in the original model converges to the critical value of the limiting process (respectively, the contact process).

For the two-stage contact process, we can show that there are at most two extremal invariant measures: the trivial one, and a non-trivial upper invariant measure that appears above the critical value. This is achieved using known techniques for the contact process. We can show complete convergence, from any initial configuration, to a combination of these measures that is given by the survival probability. This, and some additional results, are in response to the questions posed by Krone in his original paper [30] on the model.

We then generalize these ideas to develop a theory of additive growth models. In particular, we show that any additive growth model, having any number of types and interactions, will always have a dual process that is also an additive growth model. Under the additional technical condition that the model preserves positive correlations, we can then harness existing techniques to conclude existence of at most two extremal invariant measures, as well as complete convergence.