

Leavitt path algebras
Something for everyone:
algebra, analysis, graph theory, number theory

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The rings studied by students in most first-year algebra courses turn out to have what's known as the "Invariant Basis Number" property: for every pair of positive integers m and n , if the free left R -modules ${}_R R^m$ and ${}_R R^n$ are isomorphic, then $m = n$. For instance, the IBN property in the context of fields boils down to the statement that any two bases of a vector space must have the same cardinality. Similarly, the IBN property for the ring of integers is a consequence of the Fundamental Theorem for Finitely Generated Abelian Groups.

In seminal work completed the early 1960's, Bill Leavitt produced a specific, universal collection of algebras which fail to have IBN. While it's fair to say that these algebras were initially viewed as mere pathologies, it's just as fair to say that these now-so-called *Leavitt algebras* currently play a central, fundamental role in numerous lines of research in both algebra and analysis.

More generally, from any directed graph E and any field K one can build the *Leavitt path algebra* $L_K(E)$. In particular, the Leavitt algebras arise in this more general context as the algebras corresponding to the graphs consisting of a single vertex. The Leavitt path algebras were first defined in 2004. These were motivated by a related construction of a class of C^* -algebras, so called *graph C^* -algebras*, a topic in which there is significant expertise within the U. Victoria faculty. As of 2013 the subject has matured well into adolescence, currently enjoying a seemingly constant opening of new lines of investigation, and the significant advancement of existing lines. I'll give an overview of some of the work on Leavitt path algebras which has occurred in their first 8-plus years of existence, as well as mention some of the future directions and open questions in the subject.

There should be something for everyone in this presentation, including and especially algebraists, analysts, and graph theorists. We'll also present an elementary number theoretic observation which provides the foundation for one of the recent main results in Leavitt path algebras, a result which has had a number of important applications, including one in the theory of simple groups. The talk will be aimed at a general audience; for most of the presentation, a basic course in rings and modules will provide more-than-adequate background.