University of Victoria

Programme

Faculty of GRADUATE STUDIES
Thinking Outside the box

The Final Oral Examination
for the Degree of

DOCTOR OF PHILOSOPHY
(Department of Mathematics and Statistics)

MARK SCHURCH

2005	University of Victoria	MSc
1998	Simon Fraser University	BSc

"On the Depression of Graphs"

March 1st, 2013, at 11:00 am
University Centre, Room A207a

Supervisory Committee:

Dr. Christina Mynhardt, Department of Mathematics and Statistics, UVic
(Supervisor)
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic
(Academic Unit Member)
Dr. Frank Ruskey, Department of Computer Science, UVic
(Non Unit Member)

External Examiner:

Dr. Shannon Fitzpatrick, University of Prince Edward Island

Chair of Oral Examination:

Dr. Julian Lum, Department of Biochemistry and Microbiology, UVic

Abstract

An *edge ordering* of a graph G = (V,E) is an injection $f : E \to \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. A path in G for which the edge ordering f increases along its edge sequence is called an *f-ascent*; an f-ascent is *maximal* if it is not contained in a longer f-ascent. The *depression* of G is the smallest integer k such that any edge ordering f has a maximal f-ascent of length at most k. In this dissertation we discuss various results relating to the depression of a graph. We determine a formula for the depression of the class of trees known as double spiders. A k-kernel of a graph G is a set of vertices $U \subseteq V(G)$ such that for any edge ordering f of G there exists a maximal f-ascent of length at most k which neither starts nor ends in U. We study the concept of k-kernels and discuss related depression results, including an improved upper bound for the depression of trees. We include a characterization of the class of graphs with depression three and without adjacent vertices of degree three or higher, and also construct a large class of graphs with depression three which contains graphs with adjacent vertices of high degree. Lastly, we apply the concept of ascents to edge colourings using possibly fewer than $\|E(G)\|$ colours (integers). We consider the problem of determining the minimum number of colours for which there exists an edge colouring such that the length of a shortest maximal path of edges with increasing colors has a given length.

Published Papers and Conference Proceedings:

C. M Mynhardt and M. Schurch, *The Depression of a Graph and k-Kernels*, Discussiones Mathematicae Graph Theory (accepted)

C.M. Mynhardt and M. Schurch, *A Class of Graphs With Depression Three*, Discrete Mathematics (accepted)

A. Erickson and M. Schurch, *Monomer-Dimer Tatami Tilings of Square Regions*, J. Discrete Algorithms, **16** (2012) 258-269

A. Erickson and M. Schurch, *Enumerating Tatami Mat Arrangements of Square Grids*, Combinatorial Algorithms, **LNCS 7056**, (2011) 223-, 2011.

A. Erickson, F. Ruskey, M. Schurch and J. Woodcock, *Monomer-Dimer Tatami Tilings of Rectangular Regions*, Electronic Journal of Combinatorics, **18(1)** (2011) 109-133.

C.M. Mynhardt and M. Schurch, *Paired Domination in Prisms of Graphs*, Discussiones Mathematicae Graph Theory **35** (2011), 5-23

A. Erickson, F. Ruskey, M. Schurch and J. Woodcock, *Auspicious Tatami Mat Arrangements*, Computing and Combinatorics, **LNCS 6196** (2010) 288-297.

Presentations and Poster Sessions:

On Graphs with Depression Three, CanaDAM Conference (International), University of Victoria, May 31, 2011.

Awards, Grants and Fellowships

- Charles S. Humphrey Graduate Student Award (2010)
- Andy Farquharson Award for Excellence in Graduate Student Teaching (UVic 2006)