Notice of the Final Oral Examination
for the Degree of Master of Science
of
WILLIAM THOMPSON
BSc (University of Victoria, 2014)

“Analysis of a Mollified Kinetic Equation for Granular Media”

Department of Mathematics and Statistics

Tuesday, July 12, 2016
2:00 P.M.
David Strong Building
Room C126

Supervisory Committee:
Dr. Martial Agueh, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Reinhard Illner, Department of Mathematics and Statistics, UVic (Co-Supervisor)

External Examiner:
Dr. Guillaume Carlier, Ceremade, Université Paris Dauphine

Chair of Oral Examination:
Dr. Catherine Costigan, Department of Psychology, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies
Abstract

We study a nonlinear kinetic model describing the interactions of particles in a granular medium, i.e. inelastic systems where kinetic energy is not conserved due to internal friction. Such examples of particles that fall into this category are sand, snow, ground coffee and many others. Originally studied by Benedetto, Caglioti and Pulvirenti in the one-dimensional setting (RAIRO Model. Math. Anal. Numér., 31(5): 615-641, (1997)) the original model contained inconsistencies later accounted for and corrected by invoking a mollifier (Modélisation Mathématique et Analyse Numérique, M2AN, Vol. 33, No 2, pp. 439441 (1999)). This thesis approximates the generalized model presented by Agueh (Arch. Rational Mech., Anal. 221, pp. 917-959 (2016)) with the added assumption of a spatial mollifier present in the kinetic equation. In dimension $d \geq 1$ by

$$\partial_t f + v \cdot \nabla_x f = \text{div}_v(f[\eta \nabla W] *_{(x,v)} f)$$

where $f$ is a non-negative particle density function, $W$ is a radially symmetric class $C^2$ velocity interaction potential, and $\eta \alpha$ is a mollifier. A physical interpretation of this approximation is that the particles are spheres of radius $\alpha > 0$ as opposed to the original assumption of being point-masses. Properties lost by this approximation and macroscopic quantities that remain conserved are discussed in greater detail and contrasted.

The main result of this thesis is a proof of the weak global existence. An argument utilizing the tools of Optimal Transport allows simple construction of a weak solution to the kinetic model by transporting an initial measure under the characteristic flow curves. Concluding regularity arguments and restrictions on the velocity interaction potential ascertain that global classical solutions are obtained.